Electro-excitation amplitudes of the Δ - isobar in the Skyrme model

H. Walliser and G. Holzwarth*

Fachbereich Physik, Universität-GH-Siegen, D-57068 Siegen, Germany

Abstract

Electro magnetic transition form factors for the excitation of the $\Delta 33$-resonance are evaluated in the Skyrme model. They crucially rely on rotationally induced deformations of the hedgehog soliton which are suppressed by two N_C-orders as compared to the leading parts of the isovector current. Partial photon coupling through vector mesons is included in a schematic way. Recoil corrections are approximated by a boost to the equal-velocity frame. The results for the photodecay amplitudes agree with experimental numbers and the shapes of $M1, E2, C2-$ transition form factors show essential features as observed in electro-excitation experiments.

*e-mail: holzwarth@hrz.uni-siegen.d400.de

I. INTRODUCTION

Electro magnetic transition form factors for the excitation of nucleon resonances present challenging terrain for nucleon models because they sensitively reflect the nature of the states excited by the virtual photon. For the most prominent nucleon resonance, the $\Delta(1232)$, although existing data are sparse, there is sufficient indication that the magnetic ($M1$) transition form factor $G_{M1}^{N\Delta}$ is significantly different from the elastic proton magnetic form factor G_M^P. Up to the highest measured values of momentum transfer Q^2 both form factors decrease relative to the standard dipole shape $G_D = (1 + Q^2/0.71 GeV^2)^{-2}$. However, the decrease of the transition form factor sets in much earlier and with larger slope such that near 5 GeV^2 the ratio $G_{M1}^{N\Delta}/G_D$ has dropped to about half of its value at Q^2=0 while $G_M^P/\mu^P G_D$ is still close to one. On the theoretical side a recent extensive analysis in terms of a relativized quark model [1] not only has problems with obtaining a correct value of the $M1$ transition moment at $Q^2 = 0$ but shows also a severe dependence of the shape of $G_{M1}^{N\Delta}$ on the quark wave functions and configuration mixing.

For the transverse electric ($E2$) and longitudinal ($C2$) form factors the experimental information is even more rudimentary. From the point of perturbative QCD one would expect asymptotic equality of $E2$ and $M1$ transition amplitudes [2] which would imply a change of sign in $G_{E2}^{N\Delta}$ for Q^2 higher than the values where it is presently known to be much smaller and of opposite sign relative to $G_{M1}^{N\Delta}$. There are experimental indications that this sign change occurs near $1.5 - 2\ GeV$ [3].

Solitons in effective nonlinear meson field theories present an attractive alternative for the evaluation of baryon form factors, because already in the leading classical approximation of $\mathcal{O}(N_C)$ (N_C is the number of colours) the spatial structure of currents is determined through the classical solution for the soliton profile which in the language of chiral perturbation theory sums up all multi-loop graphs without closed meson loops [4] [5]. Transition moments and form factors in leading order N_C have been very early discussed in [6] [7] [8]. However, it is evident that for these observables the $\mathcal{O}(N_C)$-approximation is not sufficient: i) In classical approximation nucleon and Δ are characterized by the same soliton profile therefore transition matrix elements can differ from diagonal matrix elements only by geometrical factors, i.e. normalized form factors coincide; ii) due to the spherical symmetry of the classical hedgehog soliton quadrupole matrix elements are zero; this implies vanishing $E2$ and $C2$ form factors; iii) longitudinal matrix elements are related to the time-component of the vector current; in the equation of continuity for the vector current the contribution of the time component is suppressed by $1/N_C^2$ as compared to the leading part of the spatial components; current conservation therefore requires solving the equations of motion consistently to $\mathcal{O}(N_C^{-1})$. These rotational contributions to the isovector form factors are of the same $\mathcal{O}(N_C^{-1})$ as the isoscalar part of the magnetic form factor which always had been included in the evaluation of nucleon magnetic properties.

It has been shown [9] that inclusion of $\mathcal{O}(N_C^{-1})$-rotational effects in the equations of motion introduces quadrupole distortion into the soliton. We show in section III that the resulting structure of the currents in terms of collective operators produces nonvanishing $\mathcal{O}(N_C^{-1})$ $E2$ and $C2$ form factors and a nonvanishing difference between elastic and transition magnetic form factors. To demonstrate the essential features of their shape we evaluate them for the most simple soliton, the skyrmion. For a realistic description of the photon-

baryon coupling the Skyrme model should be augmented by vector mesons. We include their influence in an approximate form by one common vector meson propagator.

In the most interesting region of Q^2-values above $1\ GeV^2$ the shape of the form factors is sensitive to relativistic recoil corrections. Their reliable inclusion poses a serious problem in quark models as well as in soliton models. Following [10] [11] we perform a boost of the soliton to the equal-velocity-frame. Naturally, its effect depends heavily on the kinematical mass of the soliton which (in tree approximation) exceeds the actual nucleon mass by up to a factor of two. Therefore the resulting shape of the form factors for higher Q^2 is not really reliable but only indicative of the expected behaviour. It seems that also in this respect the explicit inclusion of vector mesons appears very helpful, because it leads to a sizable lowering of the soliton mass.

In section IV we present the results of this rather simple model of nucleon and Δ-resonance for the helicity amplitudes at the photon point and the transition form factors.

II. DEFINITIONS

In this section we establish the connection between the helicity amplitudes and the $M1, E2$ and $C2$ multipole operators which may contribute to the transition from the proton to the Δ-isobar. We also define the form factor conventions used in this paper.

In order to minimize errors due to recoil, the so-called "equal-velocity" (EV) frame where the incoming nucleon and the outgoing Δ have opposite velocities

$$v_\Delta^2 = v_N^2 = v^2 = \frac{q^2}{q^2 + (M_\Delta + M_N)^2}\ , \qquad \gamma^2 = \frac{1}{1 - v^2} \tag{2.1}$$

is chosen as a convenient reference frame [1]. Here $\boldsymbol{q}$ represents the three-momentum of the virtual photon in the EV-frame

$$q^2 = \boldsymbol{q}^2 = \frac{(M_\Delta + M_N)^2}{4M_\Delta M_N}\left[(M_\Delta - M_N)^2 + Q^2\right]$$
$$q_0^2 = \boldsymbol{q}^2 - Q^2\ . \tag{2.2}$$

For elastic scattering $(M_\Delta = M_N)$ this frame reduces to the Breit frame $\boldsymbol{q}^2 = Q^2$, $q_0 = 0$ and also for Q large compared to the nucleon-Δ split we have $\boldsymbol{q}^2 \simeq Q^2$, whereas at the photon point $(Q^2 = 0)$ we obtain $q_0 = |\boldsymbol{q}| = q_\Delta = (M_\Delta^2 - M_N^2)/2\sqrt{M_\Delta M_N} = 296 MeV$.

Concerning the helicity amplitudes, it is convenient to decompose the transverse ones

$$A_{\frac{1}{2}} = A_{\frac{1}{2}}(M1) + A_{\frac{1}{2}}(E2) = -\frac{1}{2}(M1 + 3E2) \tag{2.3}$$

$$A_{\frac{3}{2}} = A_{\frac{3}{2}}(M1) + A_{\frac{3}{2}}(E2) = \sqrt{3}A_{\frac{1}{2}}(M1) - \frac{1}{\sqrt{3}}A_{\frac{1}{2}}(E2) = -\frac{\sqrt{3}}{2}(M1 - E2)$$

into their $M1$ and $E2$ contributions. Then all helicity amplitudes may be expressed by simple matrix elements

$$A_{\frac{1}{2}}(M1) = \sqrt{\frac{4\pi\alpha}{2k_\Delta}} < \Delta^+, S_3 = \frac{1}{2}|M^{M1}_{\lambda=1}|p, S_3 = -\frac{1}{2} >$$

$$A_{\frac{1}{2}}(E2) = \sqrt{\frac{4\pi\alpha}{2k_\Delta}} < \Delta^+, S_3 = \frac{1}{2}|M^{E2}_{\lambda=1}|p, S_3 = -\frac{1}{2} > \qquad (2.4)$$

$$S_{\frac{1}{2}}(C2) = \sqrt{\frac{4\pi\alpha}{2k_\Delta}} \frac{M_N}{M_\Delta} \frac{q_0}{k_\Delta} < \Delta^+, S_3 = \frac{1}{2}|M^{C2}_{\lambda=0}|p, S_3 = \frac{1}{2} >$$

$(\alpha = 1/137, k_\Delta = (M_\Delta^2 - M_N^2)/2M_\Delta)$ of the corresponding multipole operators [12]

$$M^{M1}_\lambda(q^2) = i\sqrt{6\pi}\lambda \int d^3r\, V_i^3\, j_1(qr)\, [\mathbf{Y}_{11\lambda}(\hat{\mathbf{r}})]_i$$

$$M^{E2}_\lambda(q^2) = \frac{\sqrt{10\pi}}{q} \int d^3r\, V_i^3\, [\mathbf{\nabla} \times (j_2(qr)\mathbf{Y}_{22\lambda}(\hat{\mathbf{r}}))]_i \qquad (2.5)$$

$$M^{C2}_\lambda(q^2) = -\sqrt{20\pi} \int d^3r\, V_0^3\, j_2(qr) Y_{2\lambda}(\hat{\mathbf{r}}) .$$

Note here, that for the nucleon-Δ transition only the isovector piece V_μ^a of the electromagnetic current can contribute. Finally we introduce the transition form factors

$$< \Delta^+, S_3 = \frac{1}{2}|M^{M1}_{\lambda=1}(q^2)|p, S_3 = -\frac{1}{2} >= -\frac{q}{2\sqrt{2}M_N} G^{N\Delta}_{M1}(q^2)$$

$$< \Delta^+, S_3 = \frac{1}{2}|M^{E2}_{\lambda=1}(q^2)|p, S_3 = -\frac{1}{2} >= -\frac{3q_0 q}{4\sqrt{2}} G^{N\Delta}_{E2}(q^2) \qquad (2.6)$$

$$< \Delta^+, S_3 = \frac{1}{2}|M^{C2}_{\lambda=0}(q^2)|p, S_3 = \frac{1}{2} >= -\frac{q^2}{2} G^{N\Delta}_{C2}(q^2) .$$

The normalization is chosen such that the form factors at $q^2 = 0$ are equal to the corresponding transition magnetic and quadrupole moments. The helicity amplitudes (2.3,2.4) are readily expressed by these form factors as well as the electromagnetic ratio and the ratio between the longitudinal and transverse couplings

$$\frac{E2}{M1} = \frac{1}{3}\frac{A_{\frac{1}{2}}(E2)}{A_{\frac{1}{2}}(M1)} = \frac{A_{\frac{1}{2}} - \frac{1}{\sqrt{3}}A_{\frac{3}{2}}}{A_{\frac{1}{2}} + \sqrt{3}A_{\frac{3}{2}}} , \qquad \frac{C2}{M1} = -\frac{1}{\sqrt{2}}\frac{S_{\frac{1}{2}}(C2)}{M1} = \frac{\sqrt{2}S_{\frac{1}{2}}}{A_{\frac{1}{2}} + \sqrt{3}A_{\frac{3}{2}}} \qquad (2.7)$$

(our longitudinal amplitude $S_{\frac{1}{2}}$ in eq.(2.4) differs by a factor $-1/\sqrt{2}$ from the one used in [13] [14]). The definitions of the multipole operators (2.5) and form factors (2.6) involve the components V_μ^a of the vector current in the EV - frame where the soliton is moving with velocity v from eq.(2.1). If we denote the form factors evaluated in the soliton rest frame by $\tilde{G}$ the relativistically corrected form factors G in the EV - frame are approximately obtained through the relations

$$qG^{N\Delta}_{M1}(q^2) = \frac{q}{\gamma^2}\tilde{G}^{N\Delta}_{M1}(\frac{q^2}{\gamma^2}) ,$$

$$q_0 q G^{N\Delta}_{E2}(q^2) = \frac{q_0 q}{\gamma^2}\tilde{G}^{N\Delta}_{E2}(\frac{q^2}{\gamma^2}) , \qquad (2.8)$$

$$q^2 G^{N\Delta}_{C2}(q^2) = \frac{q^2}{\gamma^2}\tilde{G}^{N\Delta}_{C2}(\frac{q^2}{\gamma^2}) .$$

It should be appreciated that this procedure for solitons has a profound basis in the Lorentz-covariance of the underlying field theory: The boosted soliton again is solution of the boosted equations-of-motion. This is in contrast to 'prescriptions' in quark cluster model approaches which try to incorporate some relativity. A deficiency lies in the fact that in the Lorentz-boost the collective position and momentum variables are treated as commuting classical variables [10]. In tree approximation, however, the whole soliton approach is basically classical, so on this level the boost appears consistent. The real problem lies, however, in the fact that quantum corrections apparently are large, especially for the soliton mass, which through the Lorentz factors enters sensitively into the behaviour of formfactors for large q^2. But at tree level we have to accept this deficiency, therefore in this respect the results can only indicate general features and have no strong predictive power.

III. MULTIPOLES IN THE SOLITON MODEL

In this section we evaluate the $M1, E2$ and $C2$ multipole operators in the most simple pseudoscalar soliton model, explicit expressions for the corresponding transition form factors (2.6) will subsequently be given.

If we insert the hedgehog ansatz

$$U = A U_0 A^\dagger \, , \qquad U_0 = e^{i\boldsymbol{\tau}\hat{\boldsymbol{r}}F(r)} \tag{3.1}$$

(rotation matrix $A \in SU(2)$, chiral angle $F(r)$) into the spatial components of the vector current we find a nonvanishing $M1$ contibution but the $E2$ contribution vanishes identically. This is because the $E2$ transition is related to a quadrupole deformation which is suppressed by $1/N_C^2$ as compared to the $M1$ transition and which is not present in the hedgehog ansatz. This deformation, caused by the soliton's rotation, has to be taken into account on the same footing as for the $C2$ transition which is related to the time component of the vector current and therefore must contain an angular velocity. For that reason we have to solve the equation of motion $\partial^\mu V_\mu^a = 0$ consistently to order $1/N_C^2$. This will be done in the following subsection.

A. Rotationally induced soliton deformations

Small (time independent) soliton deformations $\boldsymbol{\eta}$ are introduced via the ansatz

$$U = A\sqrt{U_0}e^{i\boldsymbol{\tau}\boldsymbol{\eta}/f_\pi}\sqrt{U_0}A^\dagger \, , \qquad \boldsymbol{\eta} = \hat{\boldsymbol{r}}\eta_L + \boldsymbol{\eta}_T \, . \tag{3.2}$$

The driving term for these deformations (linear in $\boldsymbol{\eta}$) stems from the centrifugal term in the lagrangian which is proportional to the angular velocity $\boldsymbol{\Omega}^R$ squared

$$L_\Omega = f_\pi \int d^3r \left\{ \left[sca_L - c_4^a \frac{1}{r^2}(r^2 F' s^2)' \right] (\hat{\boldsymbol{r}} \times \boldsymbol{\Omega}^R)^2 \eta_L - sa_T(\hat{\boldsymbol{r}}\boldsymbol{\Omega}^R)(\boldsymbol{\Omega}^R\boldsymbol{\eta}_T) \right\} \, . \tag{3.3}$$

Here and in the following we use the abbreviations $c_4^a = 1/f_\pi^2 e^2$ and

$$a_L = 1 + c_4^a(F'^2 + \frac{2s^2}{r^2}) , \quad b_L = 1 + c_4^a \frac{2s^2}{r^2} ,$$

$$a_T = 1 + c_4^a(F'^2 - \frac{2s^2}{r^2}) , \quad b_T = 1 + c_4^a(F'^2 + \frac{s^2}{r^2}) , \tag{3.4}$$

$$a_0 = 1 + c_4^a(F'^2 - \frac{s^2}{r^2}) , \quad b_0 = 1 + c_4^a \frac{s^2}{r^2} ,$$

with $s = \sin F$ and $c = \cos F$. The corresponding equation of motion is

$$h_{ab}^2 \eta_b = \frac{f_\pi}{\Theta^2}\left\{ \left[sca_L - c_4^a \frac{1}{r^2}(r^2 F' s^2)' \right] \hat{r}_a(\hat{\boldsymbol{r}} \times \boldsymbol{R})^2 - sa_T(R_a - \hat{r}_a(\hat{\boldsymbol{r}}\boldsymbol{R}))(\hat{\boldsymbol{r}}\boldsymbol{R}) \right\} , \tag{3.5}$$

where the differential operator h_{ab}^2 is obtained by expanding the adiabatic lagrangian to second order in the time independent deformations and where the angular velocities are replaced by the angular momenta $\boldsymbol{R} = -\Theta\boldsymbol{\Omega}^R$ with the moment of inertia Θ. This equation of motion (3.5) is identical to the conservation of the vector current

$$-\partial^i V_i^a = \partial_i V_i^a = \dot{V}_0^a = \frac{f_\pi^2}{\Theta^2} s^2 b_T D_{ap}(\hat{\boldsymbol{r}} \times \boldsymbol{R})_p(\hat{\boldsymbol{r}}\boldsymbol{R}) . \tag{3.6}$$

In the asymptotical region the equation of motion (3.5) may be solved analytically

$$\boldsymbol{\eta} \stackrel{r \to \infty}{=} \frac{3g_A}{16\pi f_\pi \Theta^2} e^{-m_\pi r} \left[\hat{\boldsymbol{r}}(\boldsymbol{R})^2 - \boldsymbol{R}(\hat{\boldsymbol{r}}\boldsymbol{R}) \right] , \tag{3.7}$$

in accordance with ref. [15]. The full equations induce a monopole and a quadrupole deformation

$$\eta_L = \frac{1}{6f_\pi\Theta^2} \left[2f(r)\boldsymbol{R}^2 + u(r)(\boldsymbol{R}^2 - 3(\hat{\boldsymbol{r}}\boldsymbol{R})^2) \right] , \tag{3.8}$$

$$\boldsymbol{\eta}_T = -\frac{1}{2f_\pi\Theta^2} v(r)(\boldsymbol{R} - \hat{\boldsymbol{r}}(\hat{\boldsymbol{r}}\boldsymbol{R}))(\hat{\boldsymbol{r}}\boldsymbol{R}) .$$

The resulting system of differential equations for the radial functions $f(r)$, $u(r)$ and $v(r)$ is given in the appendix and has to be solved numerically subject to the boundary conditions $f(0) = u(0) = v(0) = 0$ and

$$f(r) \stackrel{r \to \infty}{=} u(r) \stackrel{r \to \infty}{=} v(r) \stackrel{r \to \infty}{=} \frac{3g_A}{8\pi} e^{-m_\pi r} , \tag{3.9}$$

compare (3.7). The radial functions are depicted in fig.1. The rotationally induced soliton deformations (3.8) are now fixed and enter into the spatial components of the vector current (see appendix).

B. Transition form factors

First we evaluate the $M1$ transition operator (2.5) by inserting the vector current with the soliton deformations (3.8) included

$$M_\lambda^{M1}(q^2) = -\frac{3\lambda}{2}\left\{\frac{2f_\pi^2}{3}\int d^3r\, j_1\frac{s^2}{r}b_T D_{3\lambda}\right.$$

$$+f_\pi\int d^3r\, j_1\left[\left(\frac{2sc}{r}a_L\eta_L + 2c_4^a\frac{F's^2}{r}\eta_L' - sa_0\boldsymbol{\nabla}\boldsymbol{\eta}_T\right)D_{3p}(\delta_{p\lambda} - \hat{r}_p\hat{r}_\lambda)\right.$$

$$\left.\left.-\frac{s}{r}a_L D_{3p}(\hat{r}_p\eta_{T\lambda} - \eta_{Tp}\hat{r}_\lambda)\right]\right\} \tag{3.10}$$

$$= -\frac{3\lambda}{2}\left\{\frac{2f_\pi^2}{3}\int d^3r\, j_1\frac{s^2}{r}b_T D_{3\lambda}\right.$$

$$+\frac{1}{45\Theta^2}\int d^3r\, j_1\left[\frac{2sc}{r}a_L(10f - u) + 2c_4^a\frac{F's^2}{r}(10f' - u') - \frac{3s}{r}a_0 v\right]\frac{1}{2}\{D_{3\lambda},\boldsymbol{R}^2\}$$

$$\left.+\frac{1}{15\Theta^2}\int d^3r\, j_1\left[\frac{2sc}{r}a_L u + 2c_4^a\frac{F's^2}{r}u' + \frac{3s}{r}a_0 v\right]L_3 R_\lambda\right\} .$$

Due to the soliton deformations there appear three different operators $D_{3\lambda}$, $\frac{1}{2}\{D_{3\lambda},\boldsymbol{R}^2\}$, $L_3 R_\lambda$ in collective coordinate space. For that reason the elastic isovector magnetic form factor and the $M1$ transition form factor are no longer related by the model independent formula $G_{M1}^{N\Delta} = \sqrt{2}G_M^V$. Instead we obtain for the elastic isovector magnetic form factor

$$\tilde{G}_M^V(q^2) = \frac{M_N}{q}\left\{\frac{2f_\pi^2}{3}\int d^3r\, j_1\frac{s^2}{r}b_T\right. \tag{3.11}$$

$$\left.+\frac{1}{30\Theta^2}\int d^3r\, j_1\frac{s}{r}[2ca_L(5f + u) + 2c_4^a F's(5f' + u') + 3a_0 v]\right\} ,$$

and for the $M1$ transition form factor

$$\tilde{G}_{M1}^{N\Delta}(q^2) = \sqrt{2}\frac{M_N}{q}\left\{\frac{2f_\pi^2}{3}\int d^3r\, j_1\frac{s^2}{r}b_T\right. \tag{3.12}$$

$$\left.+\frac{1}{20\Theta^2}\int d^3r\, j_1\frac{s}{r}[2ca_L(10f - u) + 2c_4^a F's(10f' - u') - 3a_0 v]\right\} .$$

It is noticed that the factors which multiply the contributions of the induced components are different.

For the $E2$ transition the soliton deformations (3.8) are essential, without them the operator vanishes. Instead of inserting the rotationally induced components directly into the expression for the $E2$ multipole operator (2.5) we may employ partial integration and vector current conservation (3.6)

$$M_\lambda^{E2}(q^2) = \frac{1}{iq}\sqrt{\frac{5\pi}{3}}\int d^3r\, \left[(3j_2 - qrj_3)\partial_i V_i^3 - q^2 j_2 x_i V_i^3\right]Y_{2\lambda}$$

$$= \frac{1}{iq}\sqrt{\frac{5\pi}{3}}\int d^3r\, \left\{(3j_2 - qrj_3)\dot{V}_0^3 + f_\pi q^2 j_2 D_{3p}\right. \tag{3.13}$$

$$\left.\left[b_0(rF'c\hat{\boldsymbol{r}}\times\boldsymbol{\eta} - rs\hat{\boldsymbol{r}}\times\boldsymbol{\eta}') + c_4^a\frac{F's^2}{r}(\boldsymbol{r}\times\boldsymbol{\nabla})\eta_L\right]_p Y_{2\lambda}\right\}$$

$$= \sqrt{\frac{5\pi}{3}}\frac{q_0}{q\Theta}\int d^3r\, \left\{f_\pi^2(3j_2 - qrj_3)s^2 b_T\right.$$

$$\left.-\frac{q^2 j_2}{2}\left[b_0(rF'cv - rsv') + 2c_4^a\frac{F's^2}{r}u\right]\right\}D_{3p}(R_p - \hat{r}_p(\hat{\boldsymbol{r}}\boldsymbol{R}))Y_{2\lambda} .$$

In the last step it was noticed that the entire operator may be written as a total time derivative which in the end may be replaced by iq_0 (compare the derivation of Siegert's theorem [16]). The $E2$ transition form factor may now be computed according to (2.6)

$$\tilde{G}_{E2}^{N\Delta}(q^2) = -\frac{\sqrt{2}}{9q^2\Theta}\left\{f_\pi^2\int d^3r\,(3j_2 - qrj_3)s^2b_T \right. \tag{3.14}$$
$$\left. +\frac{q^2}{2}\int d^3r\,j_2\left[b_0(rF'cv - rsv') + 2c_4^a\frac{F's^2}{r}u\right]\right\}\ .$$

Evidently, this does not exactly coincide with the quadrupole form factor because of additional contributions from the soliton deformations.

Finally we evaluate the $C2$ transition operator (2.5)

$$M_\lambda^{C2}(q^2) = -\sqrt{20\pi}\frac{f_\pi^2}{\Theta}\int d^3r\,j_2s^2b_T D_{3p}(R_p - \hat{r}_p(\hat{\boldsymbol{r}}\boldsymbol{R}))Y_{2\lambda} \tag{3.15}$$

together with the corresponding form factor (2.6)

$$\tilde{G}_{C2}^{N\Delta}(q^2) = -\frac{\sqrt{2}}{3q^2\Theta}f_\pi^2\int d^3r\,j_2s^2b_T\ , \tag{3.16}$$

which is what naively would be considered the quadrupole form factor. It is noticed that the $E2$ and $C2$ form factors coincide in the limit $q_0 = q \to 0$ which is indeed Siegert's theorem [16].

IV. RESULTS

A. Parameters of the model

For the effective action we use the standard Skyrme model with $f_\pi = 93MeV$ and $m_\pi = 138MeV$.

- The Skyrme parameter $e = 3.86$ is chosen such that the isovector magnetic moment fits its experimental value $\mu^V = 2.35$ nuclear magnetons. With this choice eq.(3.12) together with (2.2) leads to an $M1$ transition form factor at the photon point $G_{M1}^{N\Delta}(Q^2 = 0) = 3.11$ which meets exactly the experimental value for the transition amplitude $M1 = \sqrt{\pi\alpha/k_\Delta}q_\Delta/M_N G_{M1}^{N\Delta}(Q^2 = 0)$ quoted by the particle data group [17] (see table 1).

- Nonminimal couplings to vector mesons are incorporated into a common factor

$$\Lambda(q^2) = \lambda\frac{m_V^2}{m_V^2 + q^2} + (1 - \lambda)\ , \tag{4.1}$$

($m_V = 770MeV$) to be multiplied with the pure Skyrme model form factors. The choice $\lambda = 0.55$ results in an acceptable fit (for this simple model) to the elastic magnetic proton form factor over the low momentum region. Of course, it would

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be preferable to have the resonances as genuine dynamical fields properly included. Given a lagrangian which comprises $\pi, \rho, \omega, a1$, (may be even σ) mesons and photons in chiral and gauge invariant way, this is a straightforward although very tedious task. Particularly because it is the point of this paper to emphasize the importance of rotationally induced $1/N_C$ components which deviate from the usual hedgehog form of the solitons. It is not an impossible task. But in view of the fact that such a lagrangian will contain numerous coupling terms with poorly determined coupling constants we do not consider this large effort worthwhile because the results will hinge on the choice of too many parameters. (For elastic proton form factors it has been demonstrated that with suitable parameters perfect fits are possible, see e.g. [18]). We rather decided to have only one additional parameter which allows to adjust the e.m. radii when the Skyrme parameter is fixed through the magnetic moment. The pure Skyrme model with minimal coupling cannot fit both observables simultaneously (if f_π has its exptl. value of 93 MeV). Although we are confident that the rather poor agreement shown in fig.2 for the form factors could be much improved by additional flexibility gained in the parameter space of vector meson models we prefer to demonstrate the essential features with as few parameters as possible.

- The kinematical masses which enter into the prescriptions (2.8) for the boost to the EV-frame are taken as the soliton masses obtained in tree approximation with the rotational contributions included; the above choice of parameters yields the values $M_N = 1824 MeV$ and $M_\Delta = 2051 MeV$.

B. Helicity amplitudes at the photon point

With the parameters of the model fixed we now are in a position to calculate transition amplitudes and transition form factors. The values for the transition moments given by the matrixelements of the related transition operators at $q^2 = 0$ turn out to be $\mu^{N\Delta} = 3.73$ nuclear magnetons and $Q^{N\Delta} = -.037 fm^2$. The latter value corresponds to a quadrupole moment $-.062 fm^2$ of the Δ^{++}. The above quantities should not be confused with the corresponding quantities at the photon point

$$G_{M1}^{N\Delta}(Q^2 = 0) = 3.11 \ , \quad G_{E2}^{N\Delta}(Q^2 = 0) = -.020 fm^2 \ , \quad G_{C2}^{N\Delta}(Q^2 = 0) = -.027 fm^2 \ , \quad (4.2)$$

which are considerably smaller. From the form factors at the photon point helicity amplitudes and electromagnetic ratio may be computed according to (2.3)-(2.7). The transverse amplitudes and the electromagnetic ratio are compared in table 1 with experimental data and with a recent calculation in a relativized quark model [1].

Although this comparison shows that the Skyrme model reproduces the experimental transverse amplitudes with remarkable accuracy, it should be mentioned that especially the calculated electromagnetic ratio is quite sensitive to parameter changes. Therefore the precise number listed in table I for E2/M1 should perhaps not be taken too seriously, but a value of E2/M1 $\simeq -2\%$ may be accepted as a reliable estimate. On the other hand, a comparison of the calculated quantity with numbers extracted from experimental data is subject to severe ambiguities because the Δ resonance is embedded in the pion-nucleon

TABLE I. Photodecay amplitudes and electromagnetic ratio at the photon point $Q^2 = 0$.

	experiment ref. [3]	experiment ref. [17]	relativized quark model ref. [1]	Skyrme model this work
$A_{\frac{1}{2}}[10^{-3}GeV^{-\frac{1}{2}}]$	-135 ± 16	-141 ± 5	-81	-136
$A_{\frac{3}{2}}[10^{-3}GeV^{-\frac{1}{2}}]$	-251 ± 33	-257 ± 8	-170	-259
$M1[10^{-3}GeV^{-\frac{1}{2}}]$	285 ± 37	293 ± 9	188	292
$E2[10^{-3}GeV^{-\frac{1}{2}}]$	-4.6 ± 2.6	-3.7 ± 0.9	-8.6	-6.8
$E2/M1[\%]$	$-1.57 \pm .75$	$-1.5 \pm .4$	-4.6	-2.3

continuum and this background affects the exerimental data particularly in the case of the small $E2$ and $C2$ amplitudes. The question of how to extract properties of the isolated resonance is nontrivial and has lead to various theoretical investigations [19] [20] [21]. One possibility may be to compare our calculated model quantity to the value $E2/M1 = -3.5\%$ obtained by subtracting the background from the recent MAMI photo-production data [22].

With the same reservations we obtain from $G_{C2}^{N\Delta}$ (4.2) for the longitudinal amplitude at the photon point $S_{\frac{1}{2}} = .011GeV^{-\frac{1}{2}}$ which corresponds to a ratio $C2/M1 = -2.7\%$ comparable in size to the electomagnetic one. This ratio is depicted in Fig.4 as a function of Q^2.

Clearly, a complete calculation of the $\gamma N \to \pi N$ reaction which includes the coupling of the continuous πN-background with the bound Δ resonance would be highly desirable for obtaining complex helicity amplitudes for the $N\Delta$ transition. Although it is another conceptual advantage of soliton models (as compared to quark models) that they naturally contain all the ingredients necessary for such a calculation, the efforts required go far beyond the adiabatic procedures described in [29] [30] because, as we have stressed here, it is just the non-adiabatic rotational effect in the soliton profiles and in the interaction terms which are crucial for these amplitudes.

C. Transition form factors of the Δ resonance

In Fig.2 we compare the elastic magnetic proton form factor G_M^P and the $M1$ transition form factor $G_{M1}^{N\Delta}$ as functions of Q^2 with experimental data. The difference of the two form factors (apart from a less important isoscalar contribution to the elastic proton form factor) is essentially due to the soliton deformations induced by the collective rotation which yield different matrixelements for nucleon states and for nucleon and Δ states, respectively (3.11,3.12). This difference appears with the correct sign although its size is somewhat underestimated. It has a simple geometrical interpretation, namely the spatial distribution of densities where Δ-states are involved extend further out to larger radii because of the centrifugal forces and consequently these form factors fall off more rapidly as compared to the corresponding nucleon form factors. The precise shape of both the elastic and the transition form factor above $Q^2 \sim 1GeV^2$ is sensitive to the choice of the kinematical

masses in the boost transformation (2.8) with (2.2) and to the (very small) values of the nonrelativistic form factors $\tilde{G}_M(q^2)$ at $q^2 \simeq (M_\Delta + M_N)^2$. Specifics of the model (e.g. a sixth-order term in the chiral lagrangian, or explicit inclusion of dynamical vector mesons) and quantum corrections [9] are known to strongly influence these features. Therefore the predictive power of a specific model in tree approximation is quite poor in this respect and can only indicate general features.

In Fig.3 we display the $E2$ (solid line) and $C2$ (dashed line) form factors. In this plot both form factors are normalized to unity at the photon point $Q^2 = 0$. It is noticed that the Q^2-dependence of these two form factors is quite different: while the $C2$-quadrupole form factor related to the time component of the vector current falls off smoothly, the $E2$ form factor related to the spatial components changes sign at $Q \simeq 2.6 GeV$. Again details of the shape above $Q^2 \sim 1 GeV^2$ depend on the choice of a specific model and on quantum corrections.

Finally in Fig.4 we compare the calculated $C2/M1$ ratio with Bonn, NINA, and DESY results taken from ref. [31]. In contrast to the new Bonn data point at low momentum transfer (on the very left) our calculation seems to suggest much smaller values in magnitude comparable to those of the $E2/M1$ ratio. However, again we should remember here, that comparison of the model results to the quoted data is problematical.

V. SUMMARY

In this note we consider the $\mathcal{O}(N_C^{-1})$ corrections induced by the rotation of the classical hedgehog in isospace. They are crucial for conservation of the vector current to $\mathcal{O}(N_C^{-2})$ and therefore allow for an evaluation of magnetic, electric and longitudinal transition formfactors and moments consistent to that order. The essential features emerge already in the most simple Skyrme model. However, for reasonable agreement with the elastic proton magnetic form factor the Skyrme model has to be augmented by a partial photon-vectormeson coupling and relativistic recoil corrections. For these ingredients we have used only very rough approximations; they could be replaced by more involved techniques.

Apart from pion decay constant $f_\pi = 93 MeV$, pion mass $m_\pi = 138 MeV$, vector meson mass $m_V = 770 MeV$, taken at their physical values, the model then contains two parameters: the Skyrme constant e and a mixing parameter λ which allows the coupling to the photon field to be partially mediated through vector mesons. We use e to fit the isovector magnetic moment of the nucleon to its experimental value, and λ to adjust the elastic proton magnetic form factor to the standard dipole fit. All calculations are done in tree approximation and we could argue that quantum corrections expected for these oservables are absorbed into the choice of these two parameters. All results about transition moments and form factors then are free of additional parameters.

Comparing with very sophisticated and extremely tedious recent calculations [27] [28] [1] in quark bag and cluster models the essential results for the nucleon-Δ transition in this rather simple soliton model are remarkable:

- The M1 transition moment and both transverse amplitudes at photon point agree with the presently observed values within the experimental uncertainties. The rather

sensitive ratios $E2/M1$ and $C2/M1$ are obtained as -2.3% and -2.7%, respectively, for the chosen parameter set.

- The M1 transition form factor decreases significantly faster as function of Q^2 than the elastic magnetic form factor.

- There is a sign change predicted in the E2 transition form factor around 2-3 GeV^2. Its precise location is not very well defined in this calculation because it sensitively depends on the kinematical mass in the boost transformation, which is subject to large loop corrections. The shapes of the E2 and C2 transition form factors are significantly different from each other.

- The $C2/M1$ ratio tentatively follows the experimental data as a function of Q^2.

The origin of all these results are the rotationally induced monopole and quadrupole deformations of the Skyrme hedgehog. Naturally, these are crucial for the structure of the Δ resonance which in the Skyrme soliton model is an iso-rotational excited state. They are two N_C-orders down as compared to the leading parts of the currents. This is in contrast to the transition amplitudes for higher nucleon resonances like the P11(1440), D13(1520), F15(1680), etc. which correspond to time-dependent small-amplitude fluctuations of the Skyrme hedgehog and therefore are suppressed only by one N_C order. Their photo-excitation amplitudes have been evaluated previously in different versions of the soliton model without [29] and with [30] inclusion of vector mesons.

APPENDIX A:

Here we list the differential equations for the radial functions $f(r), u(r)$ and $v(r)$ which enter the monopole and quadrupole deformations

$$\eta_L = \frac{1}{6f_\pi \Theta^2}\left[2f(r)\boldsymbol{R}^2 + u(r)(\boldsymbol{R}^2 - 3(\hat{\boldsymbol{r}}\boldsymbol{R})^2)\right],$$

$$\boldsymbol{\eta}_T = -\frac{1}{2f_\pi \Theta^2}v(r)(\boldsymbol{R} - \hat{\boldsymbol{r}}(\hat{\boldsymbol{r}}\boldsymbol{R}))(\hat{\boldsymbol{r}}\boldsymbol{R}) \tag{A1}$$

induced by the soliton's rotation. We use the abbreviations (3.4) and the longitudinal and transverse potentials

$$V_L = \frac{2(c^2 - s^2)}{r^2} + m_\pi^2 c - \frac{2c_4^a}{r^2}\left[F'^2(c^2 - s^2) + \frac{s^2}{r^2} + 2F''sc - \frac{4s^2c^2}{r^2}\right]$$

$$V_T = -(F'^2 + \frac{2s^2}{r^2}) + m_\pi^2 c - \frac{c_4^a}{r^2}\left[F''sc + F'^2(2 + s^2) - \frac{2s^2c^2}{r^2}\right]. \tag{A2}$$

The differential equation for the monopole deformation $f(r)$ then becomes

$$-\frac{1}{r^2}(r^2 b_L f')' + V_L f = 2f_\pi^2\left[sca_L - c_4^a \frac{1}{r^2}(r^2 F' s^2)'\right], \tag{A3}$$

and similarly those for the quadrupole deformation $u(r)$ and $v(r)$

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$$-\frac{1}{r^2}(r^2 b_L u')' + \frac{6b_0}{r^2}u + V_L u - \frac{6cb_L}{r^2}v + \frac{3c_4^a}{r^2}\left((F'sv)' + F''sv\right) = 2f_\pi^2\left[sca_L - c_4^a\frac{1}{r^2}(r^2 F' s^2)'\right]$$

$$-\frac{1}{r^2}(r^2 b_0 v')' + \frac{6b_T}{r^2}v + V_T v - \frac{4cb_L}{r^2}u + \frac{2c_4^a}{r^2}\left(F''su - F'su'\right) = 2f_\pi^2 sa_T \ . \tag{A4}$$

With these rotationally induced soliton deformations included the vector current $V_i^3 = D_{3p}\tilde{V}_i^p$ in the intrinsic system becomes

$$\tilde{V}_i^p = f_\pi^2 \frac{s^2}{r} b_T \varepsilon_{p\ell i}\hat{r}_\ell$$

$$+f_\pi b_T\left[s\hat{\boldsymbol{r}} \times \partial_i \boldsymbol{\eta}_T - F'c\hat{r}_i\hat{\boldsymbol{r}} \times \boldsymbol{\eta} + \frac{s(1-c)}{r}\hat{\boldsymbol{r}}(\hat{\boldsymbol{r}} \times \boldsymbol{\eta})_i - sc\partial_i\hat{\boldsymbol{r}} \times (\boldsymbol{\eta}_T + 2\hat{\boldsymbol{r}}\eta_L)\right]_p$$

$$+f_\pi c_4^a(F'^2 - \frac{s^2}{r^2})\left[F'c\hat{r}_i\hat{\boldsymbol{r}} \times \boldsymbol{\eta} - s\hat{r}_i\hat{\boldsymbol{r}} \times \boldsymbol{\eta}'\right]_p + 2f_\pi c_4^a\left[\frac{s}{r}\boldsymbol{\nabla}\boldsymbol{\eta}_T + F'\eta_L' + \frac{sc}{r^2}\eta_L\right]\frac{s^2}{r}\varepsilon_{p\ell i}\hat{r}_\ell$$

$$+f_\pi c_4^a\frac{s^2}{r^2}\left[(F'c - \frac{s}{r})\hat{r}_i(\hat{\boldsymbol{r}} \times \boldsymbol{\eta}) - F'\hat{r}_i(\boldsymbol{r} \times \boldsymbol{\nabla})\eta_L - \frac{s}{r}(\boldsymbol{r} \times \boldsymbol{\nabla})\eta_{Ti} - s\hat{\boldsymbol{r}} \times \partial_i \boldsymbol{\eta}_T\right]_p \ . \tag{A5}$$

With the differential equations (A3,A4) it is straightforward to verify that the vector current $\partial_i V_i^a = \dot{V}_0^a$ (3.6) is conserved.

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FIG. 1. Radial functions $f(r)$ of the monopole deformation and $u(r)$ and $v(r)$ of the quadrupole deformation induced by the soliton's rotation (full lines). For comparison the asymptotical function is also plotted (dashed line).

FIG. 2. Proton magnetic form factor divided by the standard dipole $\mu^P G_D$ and magnetic form factor for the $N\Delta$ transition also divided by the standard dipole G_D and normalized to one as obtained from a Skyrme model with $e = 3.86$ and vector meson coupling $\lambda = 0.55$. The data for the proton form factor are from the compilations of [23] - [26] , those for the transition form factor are referenced in [1].

FIG. 3. Electric and scalar form factors for the $N\Delta$ transition divided by the standard dipole G_D and normalized to one as obtained from a Skyrme model with $e = 3.86$ and vector meson coupling $\lambda = 0.55$.

FIG. 4. Ratio $C2/M1$ between the longitudinal and transverse couplings to the Δ resonance. The experimental data are taken from ref. [31].